EQUITY EXECUTION STRATEGIES

Street Smart

Issue 39 United States | September 30, 2009

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Related Analysis:

Rodella & Sofianos, **Estimating and capturing short-term alpha**, *Street Smart,* Issue 23, March 31, 2005

Cai & Sofianos, **Multi-day executions**, *Journal of Trading,* Summer 2006

Goldman Sachs Equity Execution Strategies

OPTIMAL PARTICIPATION RATES AND SHORT-TERM ALPHA

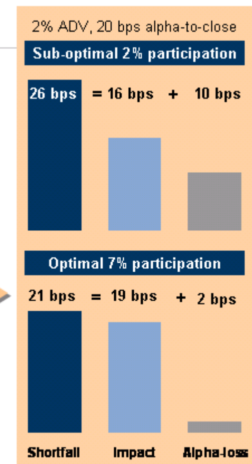
One of the most important decisions a trader has to make is how aggressively to execute. The right execution aggressiveness depends critically on short-term alpha.[1] In this issue of *Street Smart*, we derive the trader's optimal execution aggressiveness for different short-term alpha estimates. In our analysis, the trader optimizes execution aggressiveness by choosing the participation rate that minimizes execution shortfall. Shortfall has two components, liquidity impact and alpha loss. Increasing the participation rate speeds-up executions and reduces alpha loss but increases impact. **The optimal participation rate balances the reduced alpha loss of faster executions against the higher impact.**[2]

Exhibit 1. Optimal participation rates and short-term alpha[a]

Large-cap stock
9:30 start time, must fill by 16:00

Expected alpha-to-close	Order size as % of ADV: 0.25 %	0.5 %	1 %	2 %	4 %
0 bps	0.25 %	0.5 %	1 %	2 %	4 %
5 bps	1 %	1 %	1 %	2 %	4 %
10 bps	1 %	2 %	3 %	5 %	4 %
20 bps	2 %	3 %	5 %	7%	13 %
30 bps	2 %	4 %	5 %	10 %	15 %
50 bps	3 %	4 %	7 %	15 %	20%
Order size in shares	29,000	59,000	118,000	236,000	472,000

All-day executions, possible corner solutions
Minimized shortfall



a. Based on Goldman Sachs trading cost model impact estimates using input values for the week of May 11, 2009.

Source: Goldman Sachs Equity Execution Strategies

Exhibit 1 shows the optimal participation rates for various order sizes and different values of expected alpha-to-close (our short-term alpha measure). In Exhibit 1, the trader receives an order in a large-cap stock at 9:30 and must completely fill by 16:00. Consider an order 2 percent of ADV when the trader expects 20 bps alpha-to-close.[3] The optimal participation rate in this case is 7 percent and the trader minimizes execution shortfall at 21 bps. Had the trader chosen 2 percent participation, the shortfall would be 26 bps. **By choosing the optimal participation rate, therefore, the trader saves 5 bps.**

The bar charts in Exhibit 1 explain the 5 bps saving. At the optimal 7 percent participation, the impact is 19 bps and alpha loss 2 bps. At the sub-optimal 2 percent participation, the impact is 16 bps and alpha loss 10 bps. The more aggressive execution, therefore, reduces alpha loss by 8 bps but increases impact by only 3 bps.

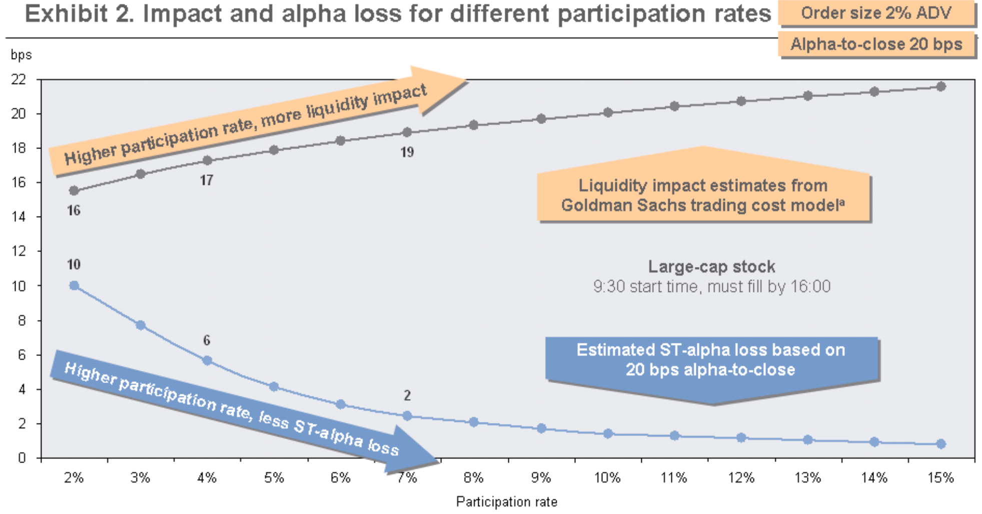
In Exhibit 1, when alpha-to-close is zero and since the order must fill by day's end, the trader will minimize shortfall by spreading the execution over the whole day. In this case, the optimal participation rate equals the order size as percent of day's volume. With order size 4 percent ADV, for example, the optimal participation rate is 4 percent. This 4 percent participation rate may be a "corner" solution in the sense that the trader could further reduce shortfall by spreading the execution over several days.[4] Corner solutions should alert traders to consider multi-day executions. We extend our derivation of the optimal participation rate to allow for multi-day executions.

ESTIMATING THE OPTIMAL PARTICIPATION RATE

In our analysis, the trader's objective is to choose the participation rate that minimizes execution shortfall. We define participation rate as the order size relative to the expected trading volume over the execution horizon. The participation rate determines the order's aggressiveness; higher participation reduces the execution horizon and increases liquidity impact.

Shortfall has two components: liquidity impact and short-term alpha loss (Appendix 1). **Increasing the participation rate increases impact but reduces alpha loss and the optimal participation rate balances this trade-off.** To derive the optimal participation rate, therefore, we must estimate how impact and alpha loss change as participation increases. Exhibit 2 summarizes our estimates for a large-cap stock and order size 2 percent ADV.[5] We assume the trader receives the order at 9:30 and must completely fill by day's end (16:00).[6]

Exhibit 2. Impact and alpha loss for different participation rates



a. Goldman Sachs trading cost model input values 5-day averages for the week of May 11, 2009.

Source: Goldman Sachs Equity Execution Strategies

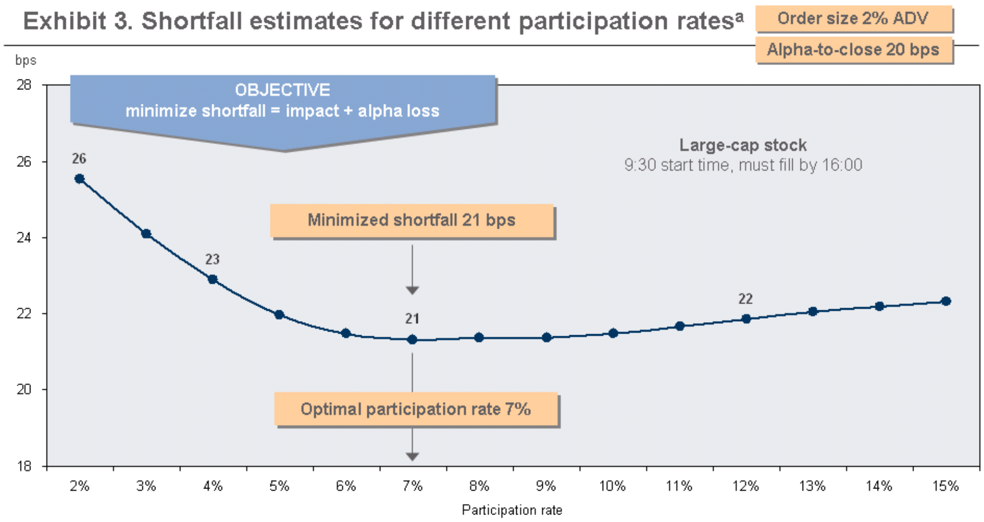
We estimate the liquidity impact using the Goldman Sachs trading cost model.[7] For each participation rate the model generates an impact estimate and the associated execution end time.[8] Since the order size is 2 percent ADV and the trader must fill by 16:00, the lowest possible participation is 2 percent (all-day execution) and the lowest impact is 16 bps. As the participation rate increases impact rises; at 4 percent participation impact is 17 bps and at 7 percent 19 bps.

We estimate the alpha loss as the alpha-to-close multiplied by an allocation factor reflecting the order's exposure to the alpha move.[9] In Exhibit 2 we assume the alpha-to-close is 20 bps. At the lowest possible 2 percent participation (all-day execution), the alpha loss is highest (10 bps when executions are spread uniformly over the day). As the participation rate increases and the execution horizon shrinks, the alpha loss decreases. At 4 percent participation the alpha loss is 6 bps and at 7 percent it is 2 bps.

The estimated shortfall is the sum of the impact and alpha loss estimates. At 4 percent participation, for example, impact is 17 bps, alpha loss is 6 bps and shortfall is 23 bps. Exhibit 3 shows how shortfall varies as the participation rate increases. At the minimum 2 percent participation the shortfall is 26 bps. As the participation rate increases, shortfall initially falls because the alpha-loss reduction more than offsets the impact increase. The shortfall continues falling up to 7 percent participation and then begins to rise. The optimal participation, therefore, is 7 percent and execution shortfall is minimized at 21 bps.[10]

Exhibit 3 shows that shortfall is flat at high participation rates (above 6 percent) and the estimated shortfall curve is not symmetric. Shortfall drops 5 bps when participation goes from 2 to 7 percent but only increases one basis point when participation goes from 7 to 12 percent. **The cost of getting it wrong, therefore, is higher if the trader is too passive (2 percent participation) than too aggressive (12 percent participation)**.

Exhibit 3. Shortfall estimates for different participation rates[a]



a. Based on Goldman Sachs trading cost model impact estimates using input values for the week of May 11, 2009.

Source: Goldman Sachs Equity Execution Strategies

In Exhibit 3 we estimate the optimal participation rate for order size 2 percent ADV and alpha-to-close 20 bps. In the same way, we estimate the optimal participation rates for all the other alpha-to-close and order size combinations in Exhibit 1. For each order size, Exhibit 1 shows how the optimal participation rate changes as the alpha-to-close increases.

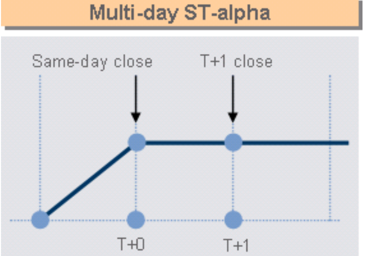
When alpha-to-close equals zero, the optimal participation rate spreads the execution throughout the allowable execution horizon from 9:30 to 16:00. In this case, the optimal participation equals order size as percent of day's volume. For order size 2 percent ADV, optimal participation is 2 percent for zero alpha-to-close, 5 percent for 10 bps alpha-to-close and 15 percent for 50 bps alpha-to-close. Appendix 2 shows the minimized shortfall associated with each of the optimal participation rates in Exhibit 1.

CORNER SOLUTIONS AND MULTIDAY EXECUTIONS

Because the order must fill by day's end, the optimal participation rate may be a corner solution. A corner solution arises when the participation rate that minimizes shortfall is below the minimum participation possible for a single-day execution. Corner solutions are more likely for large orders and low short-term (ST) alpha.

Exhibit 4 shows a corner solution for an order 8 percent ADV and 5 bps ST-alpha. At 8 percent participation, the shortfall is 34 bps but at 6 percent participation, the shortfall is even lower. Because the order size is 8 percent of day's volume, however, to achieve 6 percent participation the trader must spread the execution into the next day.[11] If the trader must fill the order in a single day, 34 bps is the lowest shortfall achievable and the optimal participation rate is 8 percent, a corner solution.

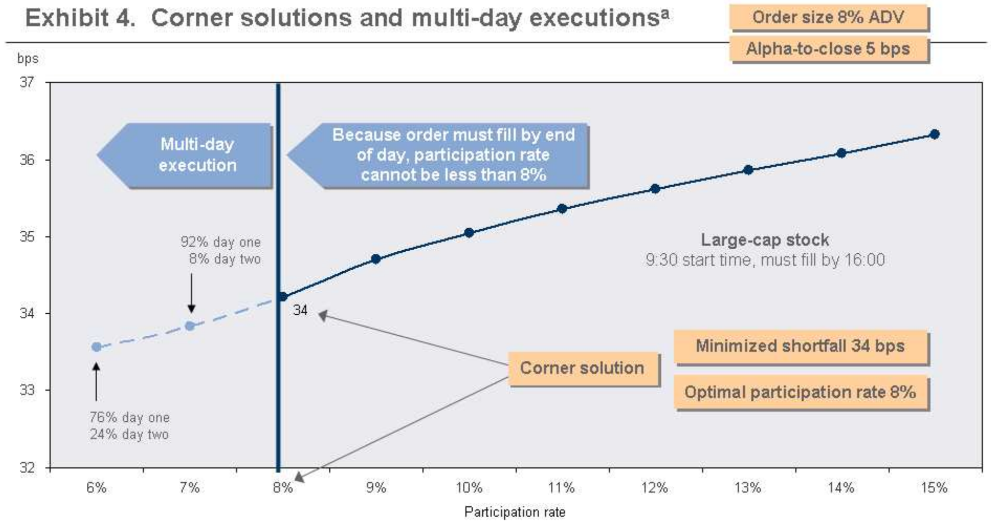
Multi-day ST-alpha



Corner solutions should alert traders to consider multi-day executions. Our approach can be extended to multi-day executions; all we need are multi-day impact and ST-alpha estimates. We get the multi-day impact estimates from the Goldman Sachs trading cost model and the multi-day alpha estimates from an analysis of the historical data. The sidebar shows the average multi-day ST-alpha pattern for Goldman Sachs algo clients. In general, most of the ST-alpha move takes place the first day and is flat for the next few days. In Exhibit 4, we derive the shortfall estimates for the 6 and 7 percent multi-day participation rates by assuming the entire 5 bps ST-alpha move takes place on the first day.

Exhibit 4 suggests that **for large orders, traders may reduce shortfall by executing part of the order on the second day, even if the entire ST-alpha move takes place on the first day.**

Exhibit 4. Corner solutions and multi-day executions[a]



a. Based on Goldman Sachs trading cost model impact estimates using input values for the week of May 11, 2009.

Source: Goldman Sachs Equity Execution Strategies

ALPHA LOSS AND EXECUTION RISK

Traders in optimizing their execution strategies face a distribution of possible ST-alpha moves over the execution horizon and must take into account both moments of the distribution:

- Average ST-alpha (first moment)
- ST-alpha volatility or execution risk (second moment)[12]

In this report, we focused on the first moment (average ST-alpha); the trader balances the increased impact of higher participation rates against reduced alpha loss. Focusing on the second moment (execution risk), requires the trader to balance the increased impact of higher participation against reduced execution risk.[13] The two approaches are complementary and the sidebar shows the trader's full optimization problem.

The full optimization

The trader chooses the participation rate to minimize:

$I + A + \lambda R$

Where:

I = Liquidity impact
A = Short-term alpha loss
R = Execution risk
λ = Trader risk aversion

(I + A = execution shortfall)

Execution risk is more relevant for small, low ST-alpha orders. For 1,000 shares in an active stock and zero ST-alpha, minimizing just the alpha loss will result in an all-day or even multi-day execution, however long it takes to avoid impact. This is unrealistic. Given the low impact cost to quickly execute a small order, most traders would opt for fast execution and reduced execution risk. When the ST-alpha is high, however, minimizing just the alpha loss will sharply reduce the execution horizon. In Exhibit 3, for example, with 20 bps alpha-to-close and 7 percent optimal participation rate, the optimized execution horizon is only 95 minutes (9:30 to 11:05). This short horizon sharply reduces execution risk and further adjusting the strategy will yield minor benefits.

ESTIMATING THE ALPHA LOSS

Our derivation of the optimal participation rate depends critically on having good estimates of how impact and alpha loss change as the participation rate increases. For participation rates below 20 percent, the Goldman Sachs trading cost model provides reliable impact estimates. As the participation rate increases beyond 20 percent, however, the slope of the cost function becomes too flat and the quality of the estimates declines.[14] We therefore avoid optimizing beyond 20 percent participation rates.

To calculate the alpha-loss we need good ST-alpha estimates.[15] ST-alpha is influenced by both the underlying investment strategy and short-term flow pressures. Traders can use three sources of information to estimate ST-alpha:

- The portfolio manager's input on the underlying investment strategy
- The traders real time "feel for the market"
- Large-sample statistical analysis of past executions.

We typically calculate the alpha-loss from the alpha-to-close which we empirically estimate from past executions.[16] From June to August 2009, for example, the average alpha-to-close for large-cap stocks in a sample of 98,000 Goldman Sachs client algo orders was 22 bps. The average order size was 2 percent ADV and the average participation rate was 7 percent, the optimal suggested by our analysis (Exhibit 3).

ST-alpha is fundamentally difficult to predict, especially order-by-order. In choosing the optimal participation rate, if traders cannot reliably differentiate between high and low ST-alpha orders, **a more practical approach is to choose the one participation rate that best fits the overall characteristics of their order flow**.

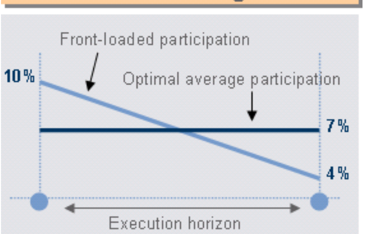
CONCLUDING COMMENTS

Because the actual ST-alpha is observable post-trade, we can use post-trade analysis to estimate what the optimal participation rate should have been and compare it with the trader's actual participation rate. Suppose over a large number of executions the trader's participation rate was an aggressive 15 percent and the actual ST-alpha was only 5 bps. We can use our optimization to recommend to the trader a less aggressive participation rate.

Determining the optimal participation rate is only the first step in optimizing an execution strategy. The trader must also determine the best way to implement the optimal participation rate. For example:

- Should the trader use market or limit orders?[17]
- Should the trader use high-touch or low-touch execution channels?[18]
- Within low-touch, which algo should the trader use?[19]

Front-loading

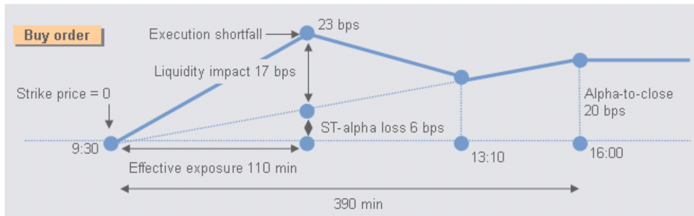


For low alpha-to-close (less than 5 bps) and low participation rates (less than 4 percent) a well-designed VWAP algo may work best. For higher participation rates, our approach most readily maps into a Participate algo where the trader explicitly sets the participation rate at the optimal level. The typical Participate algo, however, maintains a uniform participation rate over the execution horizon. When the alpha-to-close is high, the trader may benefit from a Shortfall algo that front-loads participation (see sidebar).

Our ST-alpha based optimization approach can also be used by portfolio managers to estimate the capacity of their investment strategies. Using our approach, PMs can derive the optimal execution strategy and minimized shortfall for different hypothetical AUM values. A rise in AUM will lead to an increase in both the average order size and the minimized shortfall. Capacity is exhausted when the minimized shortfall plus explicit fees multiplied by turnover equals the PMs alpha. But capacity is the subject for a future *Street Smart* report.

Appendix 1. The two components of execution shortfall

Execution shortfall = liquidity impact + short-term alpha loss



Source: Goldman Sachs Equity Execution Strategies

Calculation of ST-alpha loss

The ST-alpha loss equals the 20 bps alpha-to-close multiplied by an allocation factor reflecting the order's exposure to this move. The alpha-to-close move takes place over 390 minutes from order arrival (9:30) to close (16:00). At 4 percent participation the end time from the Goldman Sachs trading cost model is 13:10. The execution horizon is 220 minutes (9:30 to 13:10) and assuming uniform executions the order's effective exposure to the alpha move is 110 minutes (half of 220). The allocation factor in this case is 28 percent (110/390) and the ST-alpha loss is 6 bps

Appendix 2. Minimized execution shortfall estimates (in bps)[a]

Large-cap stock
9:30 start time, must fill by 16:00

		Order size as % of ADV				
		0.25 %	0.5 %	1 %	2 %	4 %
Expected alpha-to-close	0 bps	5.1	7.5	12	16	22
	5 bps	6.9	9.6	13	18	25
	10 bps	7.4	10.3	14	20	27
	20 bps	8.0	11.1	15	**21 bps**	29
	30 bps	8.3	11.6	16	22	31
	50 bps	8.8	12.3	17	24	32
	Order size in shares	29,000	59,000	118,000	236,000	472,000

Minimized shortfall for 7% optimal participation (order size 2% ADV, alpha-to-close 20 bps)

a. Based on Goldman Sachs trading cost model impact estimates using input values for the week of May 11, 2009.

Source: Goldman Sachs Equity Execution Strategies

[1] For an empirical analysis quantifying the importance of short-term alpha in determining execution shortfall see Abrokwah and Sofianos "Shortfall surprises," *Journal of Trading*, Summer 2007.
[2] An alternative approach derives the optimal execution aggressiveness by balancing the reduced execution risk of faster executions against the higher impact (see Almgren and Chriss, "Optimal execution of portfolio transactions," *Journal of Risk,* Issue 3, 2000). The two approaches, one based on ST-alpha and the other on execution risk are complementary.
[3] These figures reflect the historical averages in a sample of 98,000 Goldman Sachs client algo orders (average order size 2 percent ADV, average alpha-to-close 22 bps, large-cap stocks, June to August 2009).
[4] Formally, a corner solution arises when the constrained (must fill by day's end) optimum results in higher shortfall than the unconstrained multi-day optimum.
[5] The large-cap stock in our simulations has 12 million shares ADV and $45 price (May 2009).
[6] We ignore execution risk (or equivalently, we assume the trader is risk neutral). We discuss execution risk later in the report.
[7] For the most recent description of the Goldman Sachs trading cost model see Mie and Tierens "The enhanced Goldman Sachs shortfall model," *Street Color,* Issue 1, August 7, 2009. In our analysis, the input values are the 5-day averages for the week of May 11, 2009.
[8] Start time is always 9:30.
[9] For buy orders, alpha-to-close is the closing price minus the midquote when the trader receives the order. The alpha-to-close can be estimated from historical data (see Rodella & Sofianos "Estimating and capturing short-term alpha," *Street Smart,* Issue 23, March 31, 2005). We discuss the allocation factor in Appendix 1 (for full details see Abrokwah and Sofianos "Shortfall surprises," *Journal of Trading,* Summer 2007).
[10] The two components of the minimized shortfall are 19 bps impact and 2 bps alpha-loss. The optimized execution time is 95 minutes, 9:30 to 11:05.
[11] From 9:30 to 11:30 next day.
[12] The ST-alpha volatility can also be interpreted as measuring the uncertainty of the average ST-alpha estimate.
[13] See Rakhlin and Sofianos, "The impact of an increase in volatility on trading costs," *Journal of Trading,* Spring 2006 (Exhibit 1 summarizes the execution risk approach).
[14] There are two reasons for this. Our impact estimates are based on regression analysis of past order executions and our sample contains relatively few orders with participation rates above 20 percent. Moreover, we may have sample selection bias with the high participation rate executions in our sample: the participation rate on these executions was high because unusual amounts of counterparty liquidity were available at favourable terms. The excessive flattening of the cost curve not only underestimates the increase in impact as the participation rate increases but also makes it difficult to converge to an internal optimum.
[15] Our discussion of ST-alpha in this section summarizes the discussion in Rakhlin & Sofianos "Choosing execution strategies: VWAP or shortfall," *Journal of Trading*, Winter 2006, page 31.
[16] The main challenge with this empirical approach is to disentangle the underlying alpha move from the impact of the trade itself. We provide more details in "Estimating and capturing short-term alpha," *Street Smart*, Issue 23. To estimate the alpha loss for different participation rates we assume the alpha-to-close is linearly spread through the day. This linearity assumption may be unrealistic. We are researching the intra-day ST-alpha dynamics and can easily incorporate more realistic pattern in the optimization
[17] See Jeria, Schouwenaars & Sofianos "The all-in cost of passive limit orders," *Street Smart,* Issue 38, May 18, 2009.
[18] See Rodella & Sofianos "Order difficulty and the choice of execution strategy," *Street Smart,* Issue 22, February 22, 2005.
[19] See, for example, Rakhlin & Sofianos "Choosing execution strategies: VWAP or shortfall," *Journal of Trading*, Winter 2006.

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